Fusions and dualities for 3d T[M_3]

Tianfu fields and strings 2024

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SIMIS

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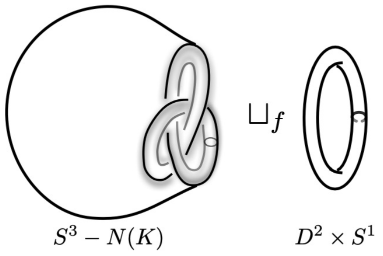
DGG construction

- 3d N=2 theories come from the compaction of 6d theories on 3-mfds.
- DGG use hyperbolic manifolds. Hyperbolic 3-mfds are decomposed into tetrahedrons. Each tetrahedron corresponds to a chiral multiplet [a conjecture].
- Hyperbolic mfds are non-compact, and exclude even basic 3-mfds, such as S^3. Gukov: DGG is a sector of a more complete story.
- Acturally, 3d-3d correspondence should be applied on compact 3-mfds.

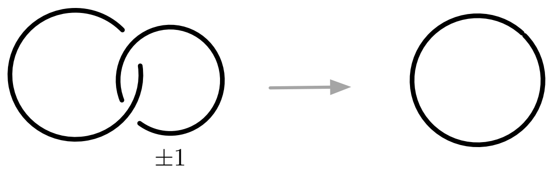
M_3 and Kirby moves

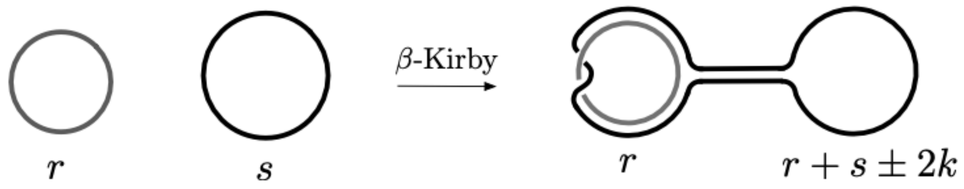
- Hyperbolic techniques are too complicated. We will only consider the basic topological structures of 3-mfds, which are described by surgeries.
- Surgery is a method to construct all compact 3-mfds. Basically, we cut out the neightboor of links, and then fill in a solid torus on each boundary:



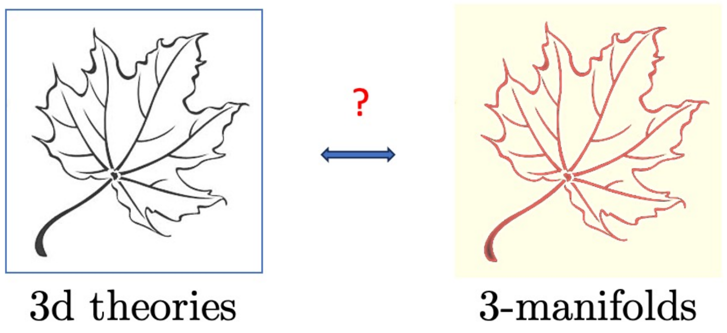
- Knots can be truned into links by Kirby moves which relate equivalent surgeries. The 3-mfds are invariant under Kirby moves.

- Kirby moves include blow up/down, and handle slides:

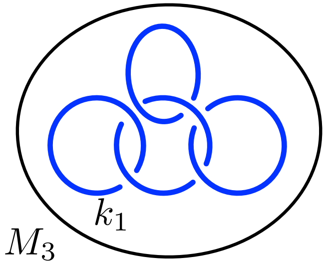


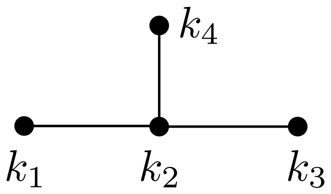


- Surgery and Kirby moves are very basic structures for 3-mfds, so they should have physical interpretations, if we believe the 3d theory T[M_3] is fully determined by the 3-mfd M_3.
- The structures of 3-mfds and 3d theories should match:



- In Gadde, Gukov, Putrov “Fivebranes and 4-mfds” [1306.4320]. 3d abelian theories T[M_3] are considered:

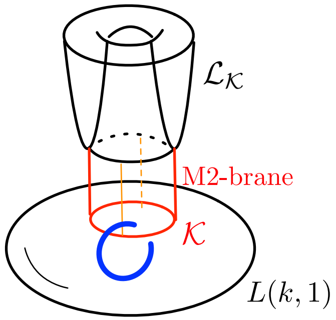




- Kirby moves are interpreted as integrating out/in vector multiplets.
- However, there is no coupled chiral multiplets in their theories. Then how to construct matters through 3-mfds. We did this by adding Ooguri-Vafa defects.

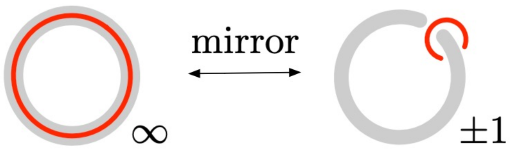
Matters and Ooguri-Vafa defects

- OV defects are non-compact Lagrangian submfds in the cotangent bundle of 3-mfds T*M3, and have the topology $\mathbb{R}^2 \times S^1$. The intersection is matter circle $\mathcal{L} \cap M_3 = S^1$.

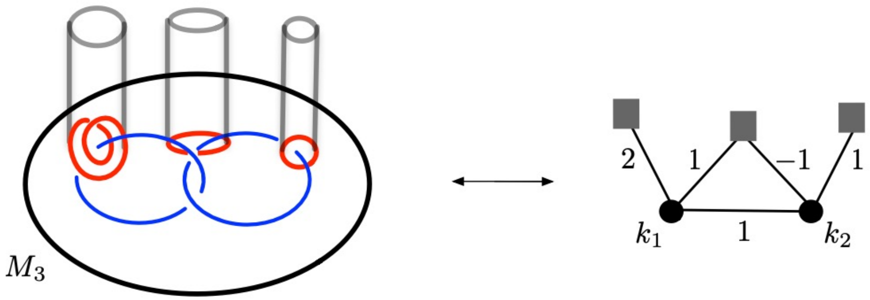


- Matter circles are these S^1, and gague circles S^1 are surgery circles.
- M/IIB duality maps this OV defect to a D5-brane. Matters come from open strings between D3-D5.

- We can use matter circles and gauge circles to represent a basic (ST-) duality $U(1)_{1/2} + 1\Phi \quad \leftrightarrow \quad 1\Phi$, which is obtained by comparing with Rolfsen twist for identical surgery.



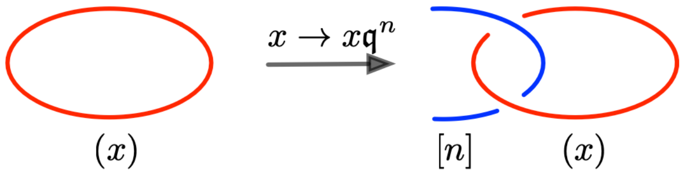
- Example:



Gauging flavor symmetries

- The method of gauging can be read off from vortex partition functions:

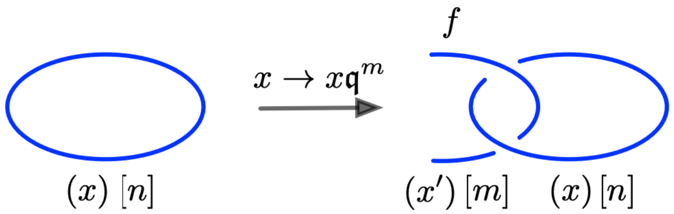
$$(x,\mathfrak{q})_\infty \xrightarrow{x\to x\mathfrak{q}^n} (x\mathfrak{q}^n,\mathfrak{q})_\infty = \frac{(x,\mathfrak{q})_\infty}{(x,\mathfrak{q})_n}$$



- Roughly, gauging a fundamental matter leads to a bi-fund. matter:

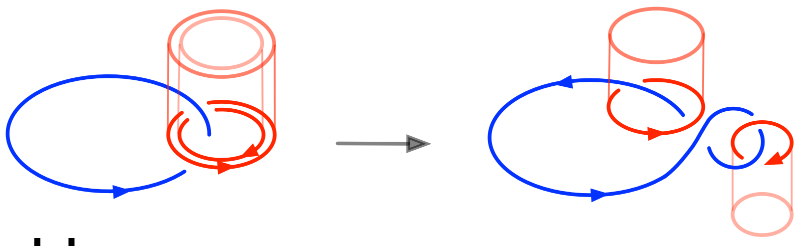
$$\bullet_{k_1} - \square \longrightarrow \bullet_{k_1} - \square - \bullet_{k_2}$$

- Gauging topological symmetry:



Flip mass parameters

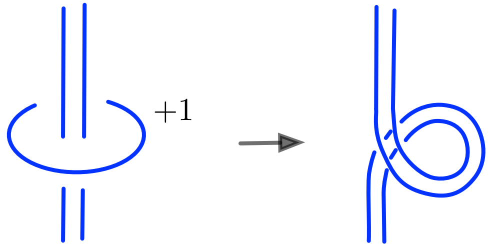
- Flipping the signs of mass parameters changes mixed Chern-Simons levels. Geometrically, it introduces a twist to gauge circles:



- The flips are described by

$$\boxed{\frac{1}{(x,\mathfrak{q})_n(\mathfrak{q}x^{-1},\mathfrak{q})_{-n}} = \frac{1}{(-\sqrt{\mathfrak{q}})^{n^2}(x/\sqrt{\mathfrak{q}})^n}}$$

- Generically, flipping a bi-fundamental matter is a Fenn-Rourke move:

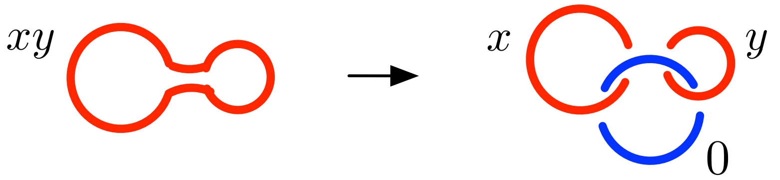


Fusion identity

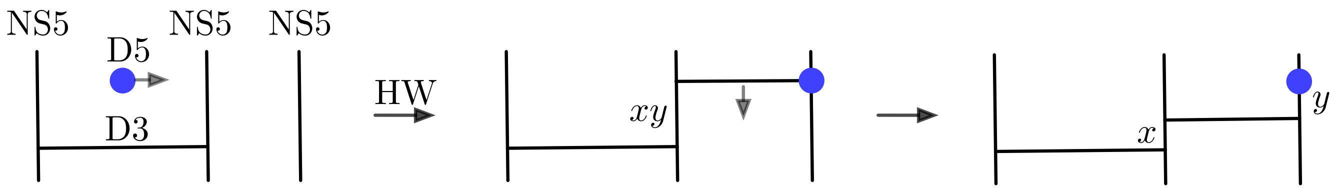
- We notice that there is a crucial identity that could derive all 3d abelian dualities that I know.

$$\frac{(xy,\mathfrak{q})_n}{(\mathfrak{q},\mathfrak{q})_n} = \sum_{k=0}^{n} \frac{(x,\mathfrak{q})_{n-k}}{(\mathfrak{q},\mathfrak{q})_{n-k}} \cdot \frac{(y,\mathfrak{q})_k}{(\mathfrak{q},\mathfrak{q})_k} \cdot x^k$$

- Fusion identity describes the connected sum of matter circles:



- Fusion identity is the operation of a gauging and a Hanany-Witten move:

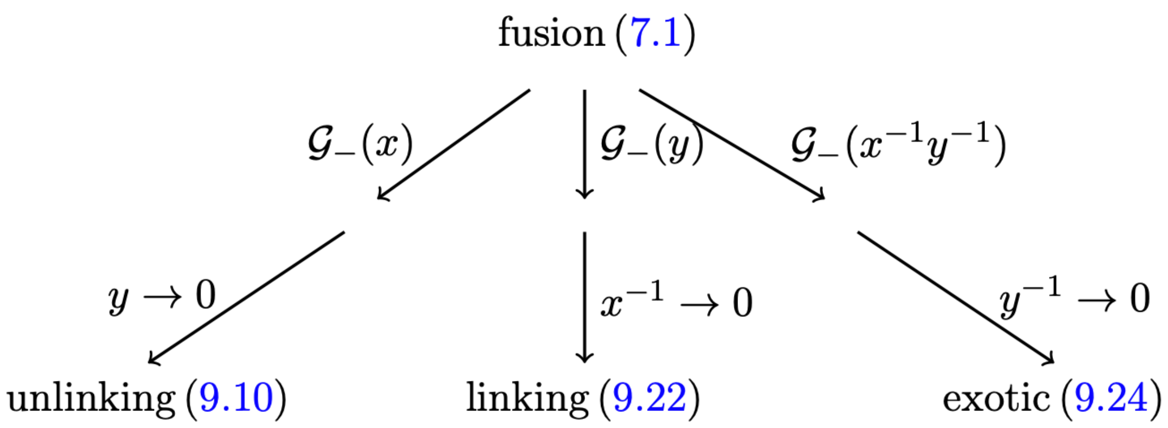


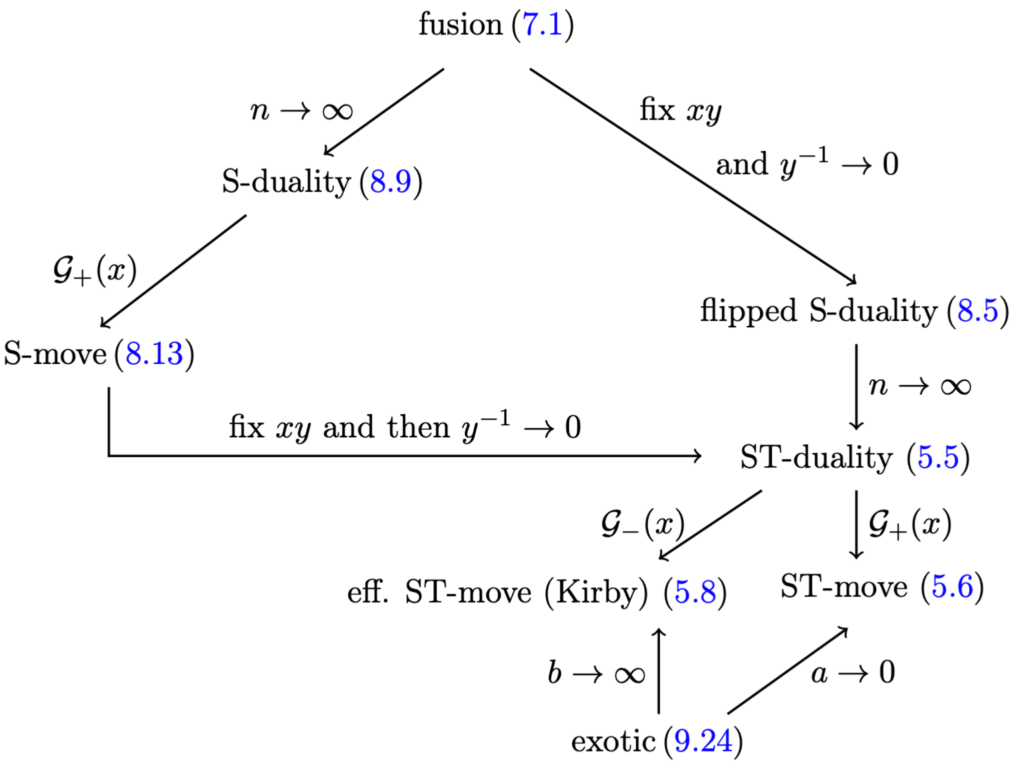
Fusion to descendent dualities

- Although fusion itself may not be an identity, but it derives the SQED-XYZ duality which contains a superpotential W=XYZ up to a gauging and a decoupling, so roughly fusion = superpotential .

- "Fusion + gauging + decoupling, large n or massless limits" lead to all dualities that I know. Since these operations also have geometric realizations, all descendent dualities inherit geometric realizations from them.

Webs of dualities from the fusion mother





Hopf algebra (work in progress)

- The fusions and flips indicate the structure of a modified Hopf algebra of quantum groups.
- Hopf algebra is defied by properties:

$$(\Delta \otimes \mathrm{id}) \circ \Delta = (\mathrm{id} \otimes \Delta) \circ \Delta$$

$$(\epsilon \otimes \mathrm{id}) \circ \Delta = (\mathrm{id} \otimes \epsilon) \circ \Delta = \mathrm{id}$$

$$(m \circ (S \otimes \mathrm{id})) \circ \Delta = \epsilon = (m \circ (\mathrm{id} \otimes S) \circ \Delta)$$

- Roughly, antipode S =flip, and the third property is the fusion identity.
- This may imply the correspondence: 3d dualities <-> extended Kirby moves <-> modified Hopf algebra.

Open questions

- How to relate fusion to R-matrix, and then prove a conjecture for knots and quivers?

- Non-abelian theories.

Thanks very much for your attention!